I Semester B.A./B.Sc. Examination, March/April 2022
(CBCS) (2014 – 15 and Onwards) (Repeaters)
MATHEMATICS (Paper – I)

Time : 3 Hours Max. Marks : 70

***Instruction** : Answer **all** questions.*

PART – A

1. Answer **any five** questions. **(5×2=10)**

   a) Define a Rank of matrix.

   b) Find the eigen value of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

   c) Find the n^{th} derivative of $\sin^2 x$.

   d) If $z = x^3 - 4x^2y + 5y^2$ find $\frac{\partial^2 z}{\partial x \partial y}$.

   e) Evaluate $\int_0^{\pi/2} \sin^6 x \, dx$.

   f) Evaluate $\int_0^{\pi/2} \sin^4 x \cos^2 x \, dx$.

   g) Find the angle between the line $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z+4}{-2}$ and the plane $x + y + z + 5 = 0$.

   h) If the two spheres $x^2 + y^2 + z^2 + 6z - k = 0$ and $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$ cuts orthogonally, find k.

PART – B

Answer **any one full** questions. **(1×15=15)**

2. a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ 3 & 2 & 6 & 7 \end{bmatrix}$ by reducing to row reduced echelon form.

   b) Find the non trivial solution of the system $x + 3y - 2z = 0$, $2x - y + 4z = 0$ and $x - 11y + 14z = 0$.

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c) Verify Cayley Hamilton Theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

OR

3. a) Reduce the matrix $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$ to normal form and find its rank.

b) Show that the system of equations $x + y + z = -3$, $3x + y - 2z = -2$, $2x + 4y + 7z = 7$ are not consistent.

c) Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 5 & -1 \\ 4 & 9 \end{bmatrix}$.

PART – C

Answer **any two full** questions : **(2×15=30)**

4. a) Find the n^{th} derivative of $\dfrac{1}{6x^2 - 5x + 1}$.

b) If $y = \sin^{-1} x$ show that $(1 - x^2)y_{n+2} - (2n + 1)\, xy_{n+1} - n^2y_n = 0$.

c) Find the n^{th} derivative of

a) $\log (5x + 4)$ b) $\cos 2x \cos 3x$.

OR

5. a) If $u = \sin^{-1}\left(\dfrac{x^2 + y^2}{x + y}\right)$ show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$.

b) Find the total derivative of u w.r.t 't' where $u = e^x \sin y$, $x = \log t$, $y = t^2$.

c) If $u = (x - y)^n + (y - z)^n + (z - x)^n$ prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$.

6. a) Find $\dfrac{df}{dt}$ where $f(x, y, z) = \log (x^2 + y^2 + z^2)$, $x = e^t$, $y = \sin t$, $z = \cos t$ by using partial differentiation.

b) If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r \cos\theta$, show that $\dfrac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta$.

c) Obtain Reduction formula for $\int \tan^n x \, dx$.

OR

7. a) Obtain Reduction formula for $\int \text{cosec}^n x \, dx$.

   b) Evaluate $\int_0^{\pi} x \cos^6 x \, dx$.

   c) Verify Leibnitz rule of differentiation under the integral sign for $\int_0^{\pi/2} \frac{dx}{\alpha(1+\cos x)}$ where α is a parameter.

PART – D

Answer **any one full** question. **(1×15=15)**

8. a) Find the equation of the plane passing through the line of intersection of the planes $2x + y + 3z - 4 = 0$ and $4x - y + 2z - 7 = 0$ and perpendicular to the plane $x + 3y - 4z + 6 = 0$.

   b) Show that the lines $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-3}{1}$ and $\frac{x-2}{2} = \frac{y-4}{1} = \frac{z-6}{3}$ are coplanar, find the equation of the plane containing them.

   c) Obtain the equation of the sphere which passes through the points (1, 0, 0) (0, 1, 0) and (0, 0, 1) and which has its centre on the plane $3x - y + z = 2$.

OR

9. a) Find the shortest distance between the skew lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-6}{4} = \frac{z-5}{5}$.

   b) Find the equation of the right circular cone whose vertex is at (2, –3, 5) axis makes an equal angles with the co-ordinate axes and the semi vertical angle is measured to be 30°.

   c) Find the equation of the right circular cylinder for which radius is 4 whose axis is the line $\frac{x-1}{2} = \frac{y-3}{-3} = \frac{z-3}{6}$.
