I Semester B.A./B.Sc. Examination, April/May 2021
(Semester Scheme) (CBCS) (F+R) (2014-15 and Onwards)
MATHEMATICS – I

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

Answer **any five** questions : **(5×2=10)**

1. a) Find the eigenvalue of the matrix $\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$.

   b) State Cayley-Hamilton theorem.

   c) Find the n^{th} derivative of $\log_e(5x-2)$.

   d) If $z = e^{\frac{x}{y}}$ find $\frac{\partial^2 z}{\partial x \partial y}$.

   e) Evaluate $\int_0^{\pi/2} \cos^5 x.\, dx$.

   f) Evaluate $\int_0^{\pi/2} \sin^3 x.\cos^4 x.\, dx$.

   g) Show that the planes $x + 2y - 3z + 4 = 0$ and $4x + 7y + 6z + 2 = 0$ are perpendicular.

   h) If the two spheres $x^2 + y^2 + 6z + z^2 - k = 0$ and $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$ cuts orthogonally, find 'K'.

PART – B

Answer **one full** question : **(1×15=15)**

2. a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 4 & -1 & 3 & -1 \end{bmatrix}$ by reducing into Echelon form.

   b) Find the non-trivial solution of the system of equations $2x - y + 3z = 0$, $3x + 2y + z = 0$ and $x - 4y + 5z = 0$.

P.T.O.

c) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 5 & -1 \\ 4 & 9 \end{bmatrix}$.

OR

3. a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$ by reducing it into normal form.

b) Solve completely the system of equations $x + 3y + 2z = 0$, $2x - y + 3z = 0$, $3x - 5y + 4z = 0$ and $x + 17y + 4z = 0$.

c) Using Cayley-Hamilton theorem find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$.

PART – C

Answer **two full** questions : **(2×15=30)**

4. a) Find the n^{th} derivative of $\dfrac{x^2}{(x+2)(2x+3)}$.

b) Find the n^{th} derivative of $\sin^3 x.\cos^2 x$.

c) If $y = (\sin^{-1}x)^2$ show that $(1 - x^2)y_{n+2} - (2n + 1)\, xy_{n+1} - n^2 y_n = 0$.

OR

5. a) If $u = (x - y)^n + (y - z)^n + (z - x)^n$ prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$.

b) State and prove Euler's theorem for homogeneous functions.

c) Find $\dfrac{du}{dt}$, if $u = x^2y^3$, $x = 2t^3$, $y = 3t^2$.

6. a) If $u = x^2 - 2y$ and $v = x + y$ find $J = \dfrac{\partial(u,v)}{\partial(x,y)}$ and $J' = \dfrac{\partial(x,y)}{\partial(u,v)}$ and verify $JJ' = 1$.

b) Verify Euler's theorem for $U = \tan^{-1}\left(\dfrac{x+y}{x-y}\right)$.

c) Obtain reduction formula for $\int \tan^n x \, . \, dx$.

OR

7. a) Obtain reduction formula for $\int \sec^n x \,.\, dx$.

b) Evaluate $\int_0^1 \frac{x^6}{\sqrt{1-x^2}} \,.\, dx$.

c) Verify the Leibnitz rule of differentiation under the integral sign for $\int_0^{\pi/2} \frac{dx}{\alpha(1+\cos x)}$, where α is a parameter.

PART – D

Answer **one full** question : **(1×15=15)**

8. a) Find the equation of the plane passing through the line of intersections of the planes $2x + y + 3z - 4 = 0$ and $4x - y + 2z - 7 = 0$ are perpendicular to the plane $x + 3y - 4z + 6 = 0$.

b) Find 'K' such that the lines $\frac{x-3}{1} = \frac{y-2}{3} = \frac{z-1}{4}$ and $\frac{x-4}{2} = \frac{y-2}{3} = \frac{Z+2}{K}$ are coplanar. For this 'K' find the plane containing the lines.

c) Find the equation of the sphere passing through the points (3, 0, 0) (0, 0, –2) and having its centre on the plane $3x + 2y + 4z - 1 = 0$.

OR

9. a) Find the shortest distance between the skew lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{1}$.

b) Find the equation of the right circular cone whose vertex is the origin, whose axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which has the semivertical angle of 30°.

c) Find the equation of the right circular cylinder for which radius 4 units and whose axis is the line $\frac{x-1}{2} = \frac{y-3}{-3} = \frac{z-3}{6}$.
