**QP – 172**

## III Semester B.A./B.Sc. Examination, March/April 2022
## (CBCS) (2015 – 16 and Onwards)
## (Semester Scheme) (Repeaters)
## MATHEMATICS (Paper – III)

Time : 3 Hours Max. Marks : 70

***Instruction :*** *Answer **all** questions.*

### PART – A

1. Answer **any five** questions. **(5×2=10)**

a) Write the order of each element of the multiplicative group $G=\{1,-1,i,-i\}$.

b) Prove that every cyclic group is abelian.

c) Prove that the sequence $\left\{\frac{3n-4}{4n+3}\right\}$ converges to $\frac{3}{4}$.

d) Test the convergence of the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots\ldots$

e) State Cauchy's mean value theorem.

f) Verify Lagrange's mean value theorem for the function $f(x) = x^2$ in [2,4].

g) Evaluate $\lim_{x\to 0}\frac{x-\sin x}{x^3}$.

h) Prove that every differentiable function is a continuous function.

### PART – B

Answer **any one** question. **(1×15=15)**

2. a) In a group G, prove that $O(a)=O(a^{-1})$.

b) State and prove Fermat's theorem in groups.

c) Prove that every group of order less than or equal to 5 is abelian.

OR

P.T.O.

3. a) In a group G, if O(a)=n $\forall a \in G$ and d = gcd(n,m) then prove that $O(a^m)=\frac{n}{d}$.

b) Prove that any two right cosets of a subgroup H of a group G are either disjoint or identical.

c) State and prove Lagrange's theorem in groups.

PART – C

Answer **any two** questions. **(2×15=30)**

4. a) If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, then prove that $\lim_{n\to\infty}(a_n + b_n) = a + b$.

b) Discuss the nature of the sequence $\{x^{1/n}\}$ where x is positive real number.

c) Examine the convergence of the sequences :

i) $\left\{\left(\frac{n+1}{n}\right)^{\frac{3n^2}{n+1}}\right\}$

ii) $\left\{\frac{1+(-1)^n n}{n+1}\right\}$

OR

5. a) Prove that a monotonic increasing sequence bounded above is convergent.

b) Show that the sequence $\{a_n\}$ where $a_1=1$ and $a_n = \sqrt{2+a_{n-1}}$ $\forall n \geq 2$ is convergent and converges to 2.

c) Test the convergence of the sequence.

i) $\{n[\log(n+1) - \log n]\}$

ii) $\left\{\left(\frac{n+1}{n}\right)^n (n+1)\right\}$

6. a) State and prove D'Alembert's ratio test for the series of positive terms.

b) Test the convergence of the series.

$$\sum \frac{1.5.9....(4n-3)}{3.7.11....(4n-1)}$$

c) Sum to infinity of the series.

$$\frac{1}{6}+\frac{1.4}{6.12}+\frac{1.4.7}{6.12.18}+....$$

OR

7. a) State and prove Cauchy's root test for the convergence of series of positive terms.

b) Discuss the convergence of the series.

$$\frac{1}{5}+\frac{\sqrt{2}}{7}+\frac{\sqrt{3}}{9}+\frac{\sqrt{4}}{11}+....$$

c) Find the sum to infinity of the series.

$$1+\frac{2^2}{1!}+\frac{3^2}{2!}+\frac{4^2}{3!}+....$$

PART – D

Answer **any one** question. **(1×15=15)**

8. a) Examine the differentiability of the function.

$$f(x)=\begin{cases} x^2-1 \text{ for } x \geq 1 \\ 1-x \text{ for } x < 1 \end{cases} \quad \text{at } x=1$$

b) State and prove Cauchy's mean value theorem.

c) Evaluate :

i) $\lim_{x \to 0} \left(\frac{1}{x}\right) \tan x$

ii) $\lim_{x \to \frac{\pi}{2}} \left(\frac{\tan 5x}{\tan x}\right)$

OR

9. a) Prove that a function which is continuous in closed interval takes every value between its bounds atleast once.

b) State and prove Rolle's theorem.

c) Expand log (secx) upto the term containing $x^4$ using Maclaurin's Series.

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