III Semester B.A./B.Sc. Examination, April/May 2021
(Semester Scheme)
(CBCS) (F+R) (2015 – 16 and Onwards)
MATHEMATICS – III

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

Answer **any five** questions : **(5×2=10)**

1. a) Find all the left cosets of the subgroup H = {0, 2, 4} of the group $(Z_6, +_6)$.

   b) Find the number of generators of the cyclic group of order 8.

   c) Show that the sequence $\left\{1-\frac{1}{n}\right\}$ is monotonically increasing sequence.

   d) State Cauchy's root test for the series of positive terms.

   e) Discuss the convergence of the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$.

   f) Discuss the continuity of the function $f(x)=\frac{1}{x^2-4}$ at $x = 2$.

   g) State Lagrange's mean value theorem.

   h) Evaluate $\lim_{x\to 0}\frac{a^x-b^x}{x}$.

PART – B

Answer **one full** question : **(1×15=15)**

2. a) If an element 'a' of a group G is of order n, then prove that $a^m = e$ for some positive integer m iff n divides m.

   b) Define a cyclic group. Show that the multiplicative group of fourth roots of unity is a cyclic group generated by < i >.

   c) State and prove Lagrange's theorem for finite groups.

OR

3. a) Prove that there is one-to-one correspondence between the set of all distinct right cosets and set of all distinct left cosets of a subgroup of a group.
   b) If G is a cyclic group of order 'd' and 'a' is a generator, then prove that $a^k (k < d)$ is also a generator of G if and only if (k, d) = 1.
   c) Prove that a finite group of prime order is cyclic and hence abelian.

PART – C

Answer **two full** questions : **(2×15=30)**

4. a) Show that the limit of a convergent sequence is unique.
   b) Discuss the nature of the sequence $\left\{x^{1/n}\right\}$.
   c) Test the convergence of the sequences
      i) $\sqrt{n^2+1}-\sqrt{n^2-1}$ ii) $\dfrac{\log(n+1)-\log n}{\sin\left(\dfrac{1}{n}\right)}$

OR

5. a) Prove that a monotonic increasing sequence bounded above is convergent.
   b) Find the limit of the sequence {0.3, 0.33, 0.333, ...}.
   c) Show that the sequence $\{a_n\}$ defined by $a_1 = 1$, $a_n = \sqrt{2+a_{n-1}}$ is convergent and converges to 2.

6. a) Discuss the nature of the geometric series $\sum_{n=0}^{\infty} x^n$.
   b) Discuss the convergence of the series $\dfrac{1}{3}+\dfrac{1.2}{3.5}+\dfrac{1.2.3}{3.5.7}+....$.
   c) Find the sum to infinity of the series $\dfrac{1}{6}+\dfrac{1.4}{6.12}+\dfrac{1.4.7}{6.12.18}+....$

OR

7. a) State and prove Raabe's test for the series of positive terms.
   b) Test the convergence of the series $\dfrac{1}{1.2.3}+\dfrac{3}{2.3.4}+\dfrac{5}{3.4.5}+....$.
   c) Find the sum to infinity of the series $\dfrac{5}{1!}+\dfrac{7}{3!}+\dfrac{9}{5!}+....$.

PART – D

Answer **one full** question : **(1×15=15)**

8. a) Show that a function which is continuous in a closed interval attain its bounds.

   b) Verify Rolle's theorem for $f(x) = \log\left(\dfrac{x^2+3}{4x}\right)$ in [1, 3].

   c) Evaluate :

      i) $\lim\limits_{x \to 0} (\cos x)^{1/x^2}$

      ii) $\lim\limits_{x \to 0} \left(\dfrac{1}{x^2} - \dfrac{1}{\sin^2 x}\right)$

OR

9. a) State and prove Cauchy's mean value theorem.

   b) Discuss the differentiability of

$$f(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right), & \text{for } x \neq 0 \\ 0 \quad , & \text{for } x = 0 \end{cases} \quad \text{at } x = 0.$$

   c) Expand $\log_e(1 + x)$ upto the term containing x^4 using Maclaurin's series.
