



SS – 344

III Semester B.A./B.Sc. Examination, November/December 2018
(Semester Scheme) (CBCS) (F + R) (2015-16 and Onwards)
MATHEMATICS – III
Mathematics

Time : 3 Hours

Max. Marks : 70

Instruction : Answer all questions.

PART – A

Answer any five questions :

(5×2=10)

1. a) Find the number of generators of the cyclic group of order 60.
- b) Find all the left cosets of $H = \{0, 4, 8\}$ in $(\mathbb{Z}_{12}, +_{12})$.
- c) Test the nature of the sequence $\{n[\log(n+1) - \log n]\}$.
- d) Examine the convergence of the series $\sum \sin\left(\frac{1}{n}\right)$.
- e) Test the convergence of the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$.
- f) Find the value of 'C' using Rolle's theorem for the function $f(x) = 8x - x^2$ in $[2, 6]$.
- g) State Lagrange's mean value theorem.
- h) Evaluate $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$.

PART – B

Answer one full question :

(1×15=15)

2. a) In a group G , prove that $O(a) = O(a^{-1}) \forall a \in G$.
- b) Find the number of generators of the cyclic group of order 8. If 'a' is one of the generator, then what are the other generators ?
- c) State and prove Euler's theorem.

OR

3. a) Any two right (left) cosets of a subgroup H of a group G are either disjoint or identical.
- b) Define cyclic group. Show that every cyclic group is abelian.
- c) If G is a finite group and H is a subgroup of G , then the order of H divides the order of G .

P.T.O.



PART - C

Answer **two full** questions :

(2x15=30)

4. a) If $\lim_{n \rightarrow \infty} a_n = a$ and $\lim_{n \rightarrow \infty} b_n = b$, then prove that $\lim_{n \rightarrow \infty} (a_n + b_n) = a + b$.
 b) Prove that a monotonically increasing sequence which is bounded above is convergent.
 c) Examine the convergence of the sequence

i) $\left\{ \left(1 + \frac{2}{n} \right)^n \right\}$

ii) $\{ \sqrt{n+1} - \sqrt{n} \}$

OR

5. a) Prove that the sequence $\left\{ \frac{3n+4}{2n+1} \right\}$ is

i) Monotonically decreasing

ii) Bounded

iii) Converges to $\frac{3}{2}$.

- b) Show that the sequence $\{a_n\}$ where $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}$ is convergent.

- c) Find limit of the sequence 0.4, 0.44, 0.444,

6. a) State and prove D'Alembert's ratio test for the series of positive terms.

- b) Examine the convergence of the series $\frac{x^2}{2\sqrt{1}} + \frac{x^3}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \dots$

- c) Sum the series to infinity $\frac{1}{6} + \frac{1.4}{6.12} + \frac{1.4.7}{6.12.18} + \dots$

OR

7. a) State and prove Leibnitz test on Alternating Series.

- b) Examine the convergence of the series $\sum \left(\frac{nx}{n+1} \right)^n$.

- c) Sum to infinity of the series $\frac{2^2}{1!} + \frac{3^2}{2!} + \frac{4^2}{3!} + \frac{5^2}{4!} + \dots$



PART - D

Answer **one full** question :

(1x15=15)

8. a) Discuss the continuity of

$$f(x) = \begin{cases} 1+x & \text{for } x < 2 \\ 5-x & \text{for } x \geq 2 \end{cases} \text{ at } x = 2.$$

b) State and prove Rolle's theorem.

c) Evaluate :

i) $\lim_{x \rightarrow 0} \left(\frac{a^x - b^x}{x} \right)$

ii) $\lim_{x \rightarrow 0} (1 + \sin x)^{\cot x}$

OR

9. a) Examine the differentiability of the function $f(x)$ defined by

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x \geq 1 \\ 1 - x & \text{for } x < 1 \end{cases} \text{ at } x = 1.$$

b) State and prove Cauchy's mean value theorem.

c) Expand $\tan x$ up to the term containing x^3 by using Maclaurin's expansion.
