SM – 371

VI Semester B.A./B.Sc. Examination, May/June 2018
(CBCS) (2016-17 and Onwards) (Semester Scheme) (Fresh + Repeaters)
MATHEMATICS – VIII

Time : 3 Hours

Max. Marks : 70

Instruction : *Answer **all** the questions/Parts.*

PART – A

Answer **any five** questions :

(5×2=10)

1. a) Evaluate $\lim\limits_{z \to -i} \dfrac{z^2 + 1}{z^6 + 1}$.

    b) Prove that $u = \dfrac{1}{2} \log (x^2 + y^2)$ is harmonic.

    c) Define an analytic function and give an example.

    d) Define bilinear transformation.

    e) Show that $f(z) = \cos z$ is analytic.

    f) State Liouvilles' theorem.

    g) Find the real root of the equation $x^3 - 9x + 1 = 0$ in (2.9, 3) by bisection method.

    h) Using Newton-Raphson method, find the real root of $x^2 + 5x - 11 = 0$ in (1, 2) in one iteration only.

PART – B

Answer **four full** questions :

(4×10=40)

2. a) Show that $\arg \left(\dfrac{z-1}{z+1} \right) = \dfrac{\pi}{4}$ represents a circle.

    b) Prove that the necessary condition for a function $f(z) = u(xy) + iv(xy)$ to be analytic is $u_x = v_y$ and $u_y = -v_x$.

        OR

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3. a) Evaluate $\lim\limits_{z \to 1+i} \left[\dfrac{z^2 - z + 1 - i}{z^2 - 2z + 2} \right]$.

 b) Show that $f(z) = ze^z$ is analytic.

4. a) Find the analytic function $f(z) = u + iv$ given that $u - v = e^x (\cos y - \sin y)$.

 b) Find the orthogonal trajectories of the family of curves

  $2e^{-x}\sin y + x^2 - y^2 = c$.

OR

5. a) If $f(z) = u + iv$ is analytic and ϕ is any differentiable function of x and y, show

  that $\left(\dfrac{\partial \phi}{\partial x} \right)^2 + \left(\dfrac{\partial \phi}{\partial y} \right)^2 = \left[\left(\dfrac{\partial \phi}{\partial u} \right)^2 + \left(\dfrac{\partial \phi}{\partial v} \right)^2 \right] | f'(z) |^2$.

 b) Show that $u = x^3 - 3xy^2$ is harmonic and find its harmonic conjugate.

6. a) Evaluate $\int\limits_{(0,\,1)}^{(2,\,5)} (3x + y)\,dx + (2y - x)\,dy$ along

  i) The curve $y = x^2 + 1$.

  ii) The line joining (0, 1) and (2, 5).

 b) State and prove fundamental theorem on algebra.

OR

7. a) Evaluate $\int\limits_C \dfrac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)\,(z-2)}\,dz$ where C is a circle $|z| = 3$.

 b) State and prove Cauchy's integral theorem.

8. a) Prove that the Bilinear transformation preserves the cross ratio.

 b) Discuss the transformation $w = z^2$.

OR

9. a) Find the bilinear transformation which maps $z = 0, - i, - 1$ on to $w = i, 1, 0$
 respectively.

 b) Show that the transformation $w = \dfrac{i - z}{i + z}$ makes the x-axis of the z-plane on

  to a circle $| w | = 1$ and the points in the half plane $y > 0$ on the points $| w | < 1$.

PART – C

Answer **two full** questions.　　　　　　　　　　　　　　　(2×10=20)

10. a) Find the root of the equation $x^3 - 4x + 1 = 0$ over (0, 1) by Regula-Falsi method.

　　b) Find the cube root of 24, correct to three decimal places by Newton-Raphson method.

OR

11. a) Solve the equation

　　　　$x + y + 54z = 110$

　　　　$27x + 6y - z = 85$

　　　　$6x + 15y + 2z = 72$ by Gauss-Seidel method.

　　b) Find the largest eigen value of the matrix and its corresponding eigen vector

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$$ by power method.

12. a) Find the solution of $\dfrac{dy}{dx} = xy$ with $y(2) = 2$ at $x = 2.1$ correct to four decimal places, using Taylor series.

　　b) Solve $\dfrac{dy}{dx} = \dfrac{y - x}{y + x}$ with $y(0) = 1$ for $x = 0.1$ by Euler's method.

OR

13. a) Solve $\dfrac{dy}{dx} = x + y$ with $y(0) = 1$ for $x = 0.1$ using Euler's modified method.

　　b) Solve $\dfrac{dy}{dx} = xy$ given $y(1) = 2$ at $x = 1.2$ by Runge-Kutta method.

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