SM – 373

IV Semester B.A./B.Sc. Examination, May/June 2018
(Semester Scheme) (Repeaters) (Prior to 2015-16) (NS) (2012-13 and Onwards)
MATHEMATICS (Paper – IV)

Time : 3 Hours Max. Marks : 100

***Instruction** : Answer **all** questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) In a ring (R, + , ·), prove that $a.0 = 0.a = 0 \ \forall \ a \in R$, where 0 is the additive identity in R.

2) Define a field.

3) Give an examples of commutative ring with zero divisors.

4) Define right ideal of a ring.

5) If (z, + , ·) be the ring of integers, define $f : z \to z$ by $f(x) = x \ \forall \ x \in z$. Prove that f is homomorphism.

6) Prove that the function $f(x, y) = \begin{cases} \dfrac{x^4 + y^4}{x^3 - y^3}, & \text{for } f(x, y) \neq (0, 0) \\ 0, & f(x, y) = (0, 0) \end{cases}$ is continuous at (0, 0).

7) Obtain the Fourier coefficient "a_0" for $f(x) = e^{-x}$ in $(-\pi, \pi)$.

8) Obtain half range sine series of $f(x) = x - 1$ in (0, 1).

9) Obtain the half range cosine series of the function $f(x) = a$ in $(0, \pi)$.

10) Write the Taylors expansion for the function f(x, y) about the point (a, b).

11) Show that the function $f(x, y) = 2x^2 - xy + y^2 + 7x$ has a minimum at (–2, –1).

12) Define Beta and Gamma function.

13) Evaluate $\int_0^\infty e^{-x} x^6 \, dx$.

14) Prove that $\overline{|(n+1)} = n\overline{|(n)}$.

15) Solve $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 0$.

16) Evaluate $\frac{1}{D^2+9}\cos 3x$.

17) Find the complementary function of

$$x^2y'' - (x^2 - 2x)y' + (x-2)y = x^2e^x,\ (x > 0).$$

18) Prove that $(2x^2 + 3x)y'' + (6x+3)y' + 2y = (x+1)e^x$ is exact.

19) Evaluate $L[e^{-2t}\cos 5t]$.

20) Evaluate $L^{-1}\left(\frac{1}{s^2+9}\right)$.

II. Answer **any three** questions : **(3×5=15)**

1) Prove that the set R = {0, 1, 2, 3, 4, 5} is a commutative ring w.r.t. $+_6$ and $\times_6$ as two ring composition.

2) Prove that the intersection of two sub-ring is a sub-ring.

3) If R be a ring and centre of R denoted by 'C' and defined as $C = \{a \in R : ab = ba\}\ \forall\ b \in R$, is a sub-ring of R.

4) Prove that the set $S = \left\{\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} : a, b \in R\right\}$ of all 2 × 2 matrices is a right ideal of the ring R of 2 × 2 matrices over z. Show that it is not a left ideal.

5) If $f : R \to R'$ be a homomorphism of R into R′. Then prove that Kernel K is an ideal of R..

III. Answer **any two** questions : **(2×5=10)**

1) Obtain the Fourier series of the function $f(x) = e^x$ in $(-\pi, \pi)$.

2) Obtain the Fourier series of the function $f(x) = |x|$ in $(-\pi, \pi)$.

3) Obtain the half-range cosine series of the function

$$f(x) = \begin{cases} 0, & \text{for } 0 < x < \frac{\pi}{2} \\ \frac{\pi}{2}, & \text{for } \frac{\pi}{2} < x < \pi \end{cases}.$$

IV. Answer **any two** questions : **(2×5=10)**

1) Expand $e^x \cos y$ near the point $\left(1, \frac{\pi}{4}\right)$ by Taylors theorem upto 2[nd] degree.

2) Test for maximum and minimum of the function $f(x, y) = x^3 + y^3 - 3xy$.

3) Prove that $\beta(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$, m, n > 0.

OR

Prove that $\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}}\,dx = \frac{1}{5005}$.

4) Show that $\int_0^1 \frac{dx}{\sqrt{1-x^n}} = \frac{\sqrt{\pi}}{n} \frac{\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}$.

OR

Show that $\int_0^1 x^5(1-x^3)^{10}\,dx = \frac{1}{396}$.

V. Answer **any four** questions : **(4×5=20)**

1) Solve $(D^2 - 4D + 4)\,y = \sin 2x$.

2) Solve $4x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} - y = 4x^2$.

3) Solve $\frac{dx}{dt} = 3x + 2y$ and $\frac{dy}{dt} + 5x + 3y = 0$.

4) Solve $y'' - 2\tan x y' - (a^2+1)y = e^x \sec x$ by changing the dependent variable.

5) Verify the equation $x^2(1+x)y'' + 2x(2+3x)y' + 2(1+3x)y = 0$ is exact and solve.

OR

Solve $y'' + y = \sec x$ by the method of variation of parameter.

VI. Answer **any three** questions : **(3×5=15)**

1) Evaluate (i) L[sin5t sint] (ii) $L[(t+2)^2]$.

2) Evaluate $L^{-1}\left[\frac{1}{(s-1)(s-2)(s-3)}\right]$.

3) If L[f(t)] = F(s), then prove that

   i) $L[f'(t)] = sF(s) - f(0)$

   ii) $L[f''(t)] = s^2F(s) - sf(0) - f'(0)$.

4) Solve using Laplace transform

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}, \text{ given } y(0) = 0,\ y'(0) = 0.$$

5) Using convolution theorem find $L^{-1}\left[\frac{1}{s(s^2+1)}\right]$.
