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END SEMESTER EXAMINATION –NOV/DEC 2025

MATHEMATICS – I SEMESTER BSC

25BSC13A: MATHEMATICS I

Time: 3 Hours **Max. Marks: 80**

Instruction: *Answer should be written completely in English.*

SECTION – A

Answer any **FIVE** questions. Each question carries **TWO** marks. **(5X2=10)**

1. State Cayley-Hamilton theorem.
2. Find the value of k such that the rank of the matrix A=$\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is two.
3. Find the n^{th} derivative of $\log(4x + 3)$.
4. If $u = x^2$, $y = v^2$, find $\frac{\partial(u,v)}{\partial(x,y)}$.
5. Write the formula to find the surface area generated by revolving the curve y=f(x) about the x-axis.
6. Evaluate $\int_0^{\frac{\pi}{2}} cos^6 x \, dx$.
7. Write the condition of orthogonality of two spheres.
8. If a right circular cone has three mutually perpendicular generators, show that the semi vertical angle is $\tan^{-1}(\sqrt{2})$.

SECTION -B

Answer any **SIX** questions. Each question carries **FIVE** marks. **(6X5=30)**

9. Find the rank of the matrix A = $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ by reducing into Normal form.
10. Verify Cayley Hamilton theorem for the matrix $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.
11. Find the n^{th} derivative of $\frac{1}{6x^2-5x+1}$.
12. If $u = x + 3y^2 - z^3$, $v = 2x^2 - yz$, and $w = 2z^2 - xy$ evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, -1, 0).
13. Evaluate $\int_0^{\frac{\pi}{2}} sin^5 \theta cos^5 \theta \, d\theta$.
14. Find the surface area of the hemisphere of radius a.
15. Find the centre and radius of the sphere $x^2+y^2+z^2-6x+8y-10z+1=0$.

16. Find the equation of the right circular cone generated when the straight line $2y + 3z = 6$, $x = 0$ revolves about z – axis.

SECTION – C

Answer any **FIVE** questions. Each question carries **EIGHT** marks. **(5X8=40)**

17. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

18. Find the inverse of the following matrix $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ by using elementary transformations.

19. If $x = r\cos\theta$, $y = r\sin\theta$ find $J = \frac{\partial(x,y)}{\partial(r,\theta)}$ and $J' = \frac{\partial(r,\theta)}{\partial(x,y)}$ and verify that $JJ' = 1$.

20. a) If $u = f(x, y)$ is a homogeneous function of x and y with degree n, then prove that
$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$.

    b) If $u = cot^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\cot u$.

21. a) Evaluate $\int_0^{\pi} x sin^7 x dx$.

    b) Evaluate $\int_0^{\pi} \frac{(1-cos\theta)}{(1+cos\theta)^2}\, sin^2\theta d\theta$.

22. Evaluate $\int_0^1 \frac{x^a-1}{logx}dx$, where a is a parameter using Leibnitz rule of differentiation under the integral sign.

23. Find the equation of the sphere which passes through the four points (1, 2, 3), (0, -2, 4), (4, -4, 2) and (3, 1, 4).

24. a) Derive the equation of the right circular cone in its standard form $x^2 + y^2 = z^2 \tan^2 \alpha$.

    b) Find the equation of right circular cone whose vertex is the origin whose axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which has semi vertical angle of 30^0.