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RESEARCH ARTICLE

Linear Programming Problems using Graph Theory

Roopa. J[1]* and Hemalatha N .C[2]

[1]Assistant Professor, Department of Mathematics (PG),St Francis De Sales College, Electronic City, Bangalore-100, Karnataka, India
[2]Associate Professor, Department of Mathematics,The Oxford College of Engineering, Bommanahalli, Bangalore-100, Karnataka, India

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***Address for Correspondence**
Roopa. J
Assistant Professor,
Department of Mathematics (PG),
St Francis De Sales College,
Electronic City, Bangalore-100,
Karnataka, India
E.Mail: roopaj3065@gmail.com

ABSTRACT

In mathematics, graph theory plays a major role in formulation of structures and to get solution for different types of real-world problems. So, graph can be formulatedby using vertices or points or nodes which are connected with an edges or lines. Graph theory can be used to solve different types of problems in the field of Social Networking, Economy, Biology, Physics, Chemistry, Computer science and many more fields. Various practical problems can be represented by using graphs. Linear programming problem is a branch of mathematical programming. By using Linear programming method, we get an accurate value of the problem. The lineal programming problems are to optimization i.e., maximization or minimization to the real-world problems. Mainly linear programming problems use to maximize the profit and to minimize the cost and waste. In this paper, we try to solve real world problems like transportation problems using Bipartite graph, diet problems and manufacturing problems graphically and get the accurate results.

Keywords: Linear Programming Problem, Bipartite Graph, Manufacturing problems, Transportation problems, Diet problems.

INTRODUCTION

A Graph G= (V, E) contains a finite set V= {v_1, v_2, v_3,…v_i} of vertices and a finite set E={e_1,e_2,….e_j} of edges. Where each edge is a pair {v_i, v_j} of vertices. The basic condition to draw a graph that they must have at least one vertex. A graph can be drawn with single vertex without an edge is called *Trivial Graph*. The reason behind edge is to connect the vertices and formulate the complete graph. In addition to that the vertices, edges cannot live in separately. The generalization of a graph in which an edge can join any number of vertices. In contrast in an ordinary graph, an edge connects exactly two vertices.

Linear programming is a branch of mathematical programming. Linear Programming (LP) is defined as the problem of optimization (i.e., maximization or minimization) of a linear function that is subject to linear equations. It mainly considered to be the most important method of optimization of different fields. It is used to obtain the most optimal solution of the problem within some variables. It composed to an objective function, linear inequalities subject to some constrains should be in the form of linear equations or in the form of inequalities. This method is used to maximize or minimize the objective function of the given mathematical model composing the set of linear inequalities depending upon some constraints (or variables) represented in the linear programming relationship.
Linear programming requires the formulation of equations or inequalities and graphically can able to solve the problems. Some particular problems can be able to solve linear programming problems.
Feasible Solution:
Feasible solution of linear programming problem is that satisfies all the variables in the LPP which we can find as a common region on the graph.

Basic Concepts of Graph theory used in Linear Programming problems:

- **Vertex:** vertex is a point or nodeon a plane where two or more-line segments meets.
- **Edges:** a line segment which connects the number of vertices or nodes.
- **Loop:** A loop is an edge who's initial and end points are same.
- **Incident:** Incident edges are edges which will be connected with the same vertex.
- **Adjacent:** if two vertices are connected with one edge is called adjacent of two vertices.
- **Degree:** The number of edges that are connected to the vertex. The degree of the loop is always two.
- **Path:** A path is any route along with no repeated edges in a graph.
- **Walk:** It is a sequence of edges and vertices in a graph.
- **Circuit:** It is a closed walk with every edge different.
- **Cycle**: A cycle is a graph in which it starts at the same vertex and end with the same vertex contains the number of edges and vertices.
- **Bipartite:** Bipartite Graph is a graph in which the vertices can be divided into two adjacent sets, such that no two vertices within the same set are adjacent.

Linear Programming using graph theory:

Linear Programming Problem:

Linear programming is a method of optimisation of operations with some linear variables. The main objective of linear programming is to maximize or minimize the numerical value. It consists of linear functions which are subjected to the variables in the form of linear equations or in the form of inequalities. Here are some examples we have solved linear programming problems graphically.

Example:

1. solve the given LPP graphically:
   maximize z=8x+y and the constraints are $x+y\leq 40$, $2x+y\leq 60$ with $x\geq 0$, $y\geq 0$[5]

Solution

The given problem is a linear programming problem.
Consider the given inequalities into equations.
p+q=40--------------(1)

2p+q=60-------------(2)

Substitute p=0 in equation (1)

q=40

The coordinate point is (0,40)

Substitute q=0 in equation (1)

p=40

The coordinatespoit is (40,0)

Substitute p=0 in equation (2)

q=60

The coordinate point is (0,60)

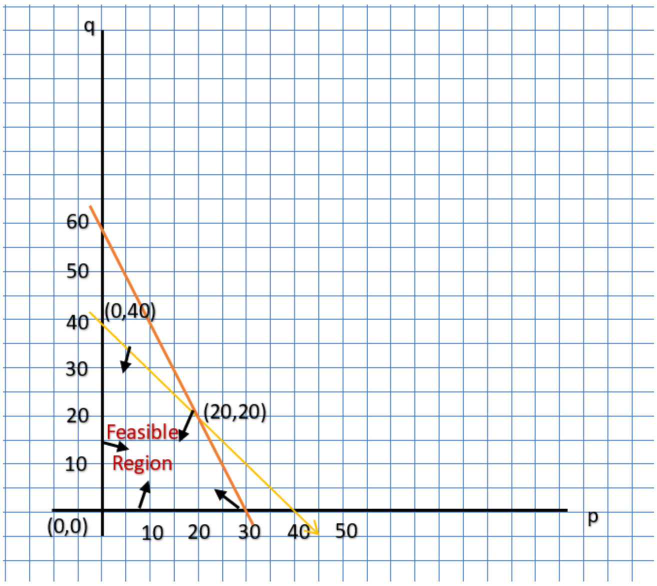
Substitute p=0 in equation (2)

2p=60

q=30

The coordinate point is (30,0)

Now, plot all the points (0,40), (40,0), (0,60) and (30,0) in the pq-plane.



Here (0,0), (0,40), (20,20) and (30,0) is a common region.
Thus, (20,20) is the intersection of two lines p+q=40 and 2p+q=60.
Then we have p=20 and q=20.
The given objective function is Max Z=8p+q at (0,0) is
Max Z=0
Max Z at (0,40) is

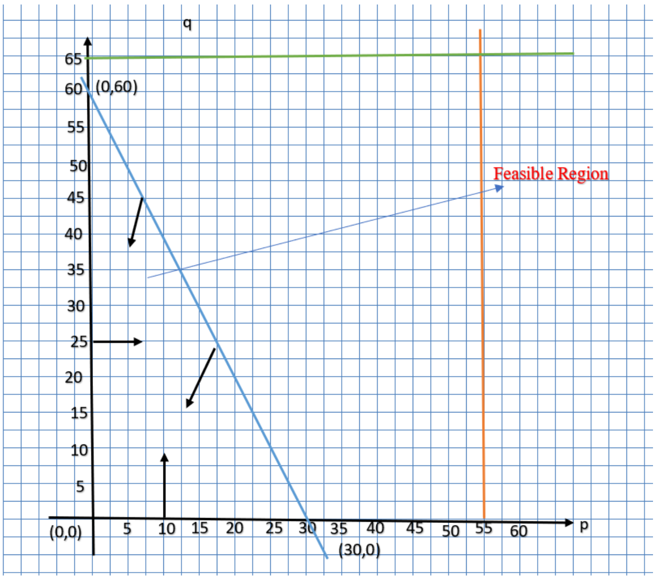
Max Z=40
Max Z at (20,20) is
Max Z=180
Max Z at (30,0) is
Max Z=240
The given problem is to maximize.
Thus, the solutions arep=30 and q=0.
Therefore, *Max Z=240.*

There are fourtypes of Linear Programming Problems.

1. Manufacturing problems
2. Diet problems
3. Transportation problems
4. Optimal Assignment Problems

Manufacturing Problems:
Manufacturing problems which deal with the production rate or net profits of manufactured problems. It can determine the number of units of different products which can must be produced and sold by a firm when each product requires a fixed manpower, machine hours, labour hour per unit of production, warehouse space per unit of the output etc., in order to make maximum profit.
Example:
1. A Manufacturing company manufactures two types of products, product P and Q. The production of these two products will not cross 25 and 35 per week respectively. According to the demand the production will takes place. For the company 4man power requires per week to produce the product P and 2 man power for the product Q. The product P makes the a profit of Rs.80 while Q makes Rs.60. Formulate the LPP.

Solution:
Given pand qare number of units for model P and Qto be produced, respectively.
The Linear Programming problem can be formulated as
Maximize (total profit) Z=80p+60q
Subject to $p \leq 55$
$q \leq 65$
$4p+2q \leq 120$, and $p,q \geq 0$
Convert all the inequalities into equalities
p=55-------(1)
q=65-------(2)
4p+2q=120------(3)
From equation (1) we get
The coordinate point is (55,0)
From equation (2) we get
The coordinate point (0,65)
Substitute p=0 in equation (3)
q=60
The coordinate point is (0,60)
Substitute q=0 in equation (3)
p=30
Now, plot the points (55,0) (0,65), (0,60) and (30,0) in the pq-plane.



Here (0,0), (30,0) and (0,60) are common region.
The given objective function is Maximum Z=80p+60 qat (0,0) is
Max Z=0
Max Z at (30,0) is
Max Z=2400
Max Z at (0,60) is
Max Z=3600
The given problem is to maximize.
Thus, the solutions are at p=0 and q=60.
Therefore, ***Max Z=3600.***

Diet Problems:
Mainly, the diet problem is to dietthe of food that should satisfy our daily nutritionalcontent at minimums. If a balanced food is taken into consideration, people's lifestyle will indeed have a promising future because there will be a benefit in definite. Mainly the people's lifespan will be increased and can prevent falling sick often and also productivity rate will increase. The diet problem is to formulate with linear equations or inequalities as a linear programme where the objective is to minimize cost and the variables are satisfy the specified nutritional requires for the body. The diet problem will insists that the quantity of calories and the amount of vitamins, minerals, fats, sodium and cholesterol in the diet. While the mathematical formulation is simple, the solution may not be tastable. The nutritional requirements which is irrespective of the for taste, so consider the output before digging into a meal from an optimal menu.

Example:
A dietician has to develop a special diet using two foods P and Q. Each pocket (containing 30g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. the diet required at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimise the amount of vitamin A in the diet? What is the minimum amount of vitamin A?[13]

Solution:

Let number of packets of food P be p.

Number of packets of food Q be q.

The Linear Programming problem can be written as

Minimize Z=6p+3q

Subject to (i) Calcium

$4p+q\geq 80$

(ii) Iron

$p+5q\geq 115$

(iii) Cholesterol

$3p+2q\leq 150$

Also, $p,q\geq 0$

Convert all the inequalities into equalities.

$4p+q=80$-------(1)

$p+5q=115$-------(2)

$3p+2q=150$-------(3)

Substitute p=0 in equation (1)

q=80

The coordinate point is (0,80)

Substitute q=0 in equation (1)

p=20

The coordinate point is (20,0)

Substitute p=0 in equation (2)

q=23

The coordinate point is (0,23)

Substitute q=0 in equation (2)

p=115

The coordinate point is (115,0)

Substitute p=0 in equation (3)

q=75

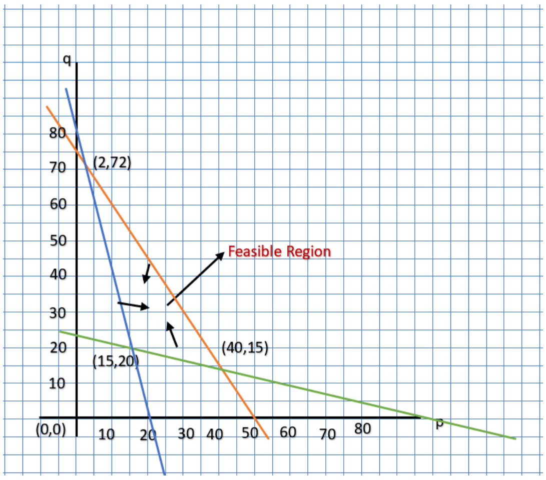
The coordinate point is (0,75)

Substitute q=0 in equation (3)

p=50

The coordinate point is (50,0)

Now, plot the points (0,80), (20,0), (0,23), (115,0), (0,75) and (50,0) in the pq-plane.



Here A (2,72), B (15,20) and C (40,15) are closed polygon.
The given objective function is Maximum Z=6p+3q at (2,72) is
Max Z=228
Max Z at (15,20) is
Max Z=150
Max Z at (40,15) is
Max Z=285

The amount of vitamin A will be minimum if 15 packets of food P and 20 packets of food Q are used.
Thus, *minimum amount of vitamin* ***A = 150 units.***

Transportation Problems:

Transportation problem is a type of linear programming problem the goods or items are carried over some set of destination from supply to the demand with minimum cost level of transportation. When both supply rate and transportation rate is equal then the problem is called balanced transportation problem. When the supply rate and the demand rate is unequal then the problem is called unbalanced transportation problem. The aim of transportation problem is to transport the goods from supply to demand in minimum transportation cost. Different techniques have been developed to solve the transportation problem. Here we solve Transportation problem using Bipartite Graph.

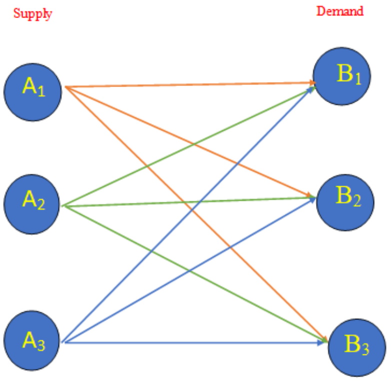
Example:

A transport company is planning to allocate owned vehicle to cities X, Y and Z. Here arethe transport table that have been prepared by managers of the company which gives the transportation cost from warehouse (Supply points) to the cities (Demand points)[9]

	X	Y	Z	Supply
1	6	8	20	150
2	7	11	11	175
3	4	5	12	275
Demand	200	100	300	

Solution

Step 1



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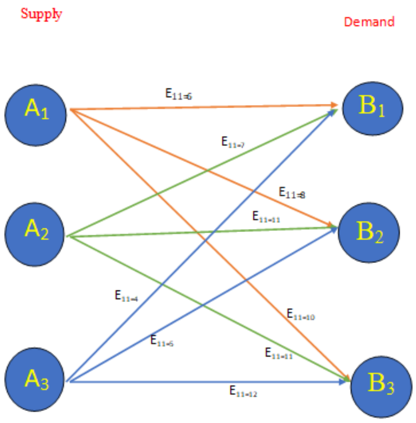

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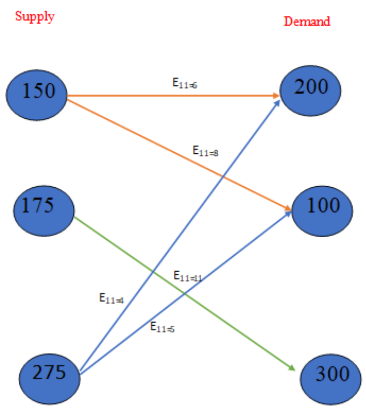

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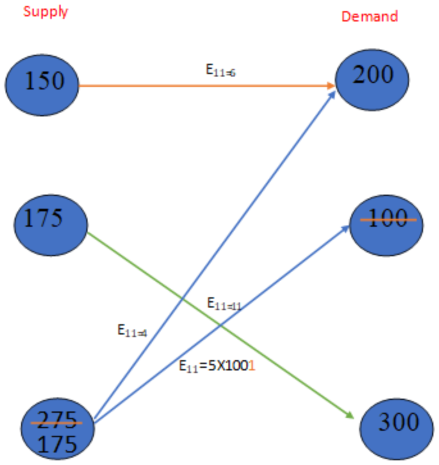
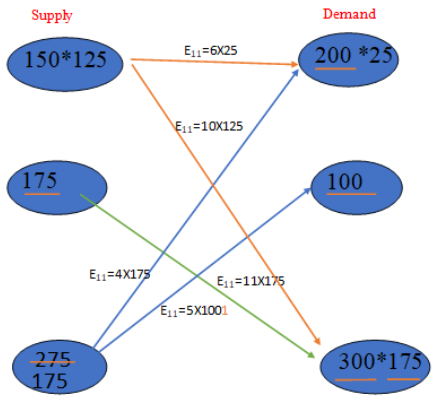


Step 5



Maximum Cost=(5x100) +(4x175) +(6x25) +(10x125) +(11x175) =4,525
Therefore, *the Optimal solution is* ***4,525.***

Applications:

Linear Programming Problems can be used in following ways:

- Personal Assignment problem solving.
- Transportation Problem solving.
- Proficiency in Operation of Dam System problem solving.
- Optimum Estimation of executive Compensation.
- Agricultural planning applications.
- Military Problem Solving.
- Production Management for determining Products.
- Marketing Management Problem solving.
- Manpower Management analysis.
- Physical distribution in Local industrial plants.

CONCLUSION

In this Paper we havetry to explain how the graph theory can be applicable in mathematics as well as linear programming problems. We have tried to solve many types of real-world problems like transportation problems, diet problems and manufacturing problems also. The Graph theory used in different types of finding maximisation or minimization of the problems. In the similar manner we can use graph theory in our day-to-day time to solve different types of problems.

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